MATHEMATICS APPLICATIONS UNIT 3
Section Two:
Calculator-assumed

SOLUTIONS

Student Number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	11	11	100	98	65
				Total	100

Instructions to candidates

1.   The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer booklet.

3.   You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4.   Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5.   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6.   It is recommended that you do not use pencil, except in diagrams.

7.   The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **eleven (11)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(6 marks)**

The weight, W_n kg, of flour produced by a mill that needs to be sent to the packing department is given by $W_{n+1} = W_n + 1.25$, $W_0 = 7.5$, where n is the number of minutes after 5 am.

(a) Complete the table below. (2 marks)

n	0	1	2	3	4	5
W_n	7.5	**8.75**	**10**	**11.25**	**12.5**	**13.75**

Solution
See table
Specific behaviours
✓ at least three values correct
✓ all values correct

(b) Calculate the weight of flour at 6 am. (2 marks)

Solution
$W_{60} = 82.5 \text{ kg}$
Specific behaviours
✓ identifies n
✓ states value

(c) At what time will the weight of flour reach 150 kg? (2 marks)

Solution
$W_{114} = 150$
$5 \text{ am} + 114 \text{ m} = 6{:}54 \text{ am}$
Specific behaviours
✓ identifies n
✓ states time

Question 9   **(13 marks)**

Agricultural researchers collected data on the amount of rainfall (x mm) and the yield of cucumbers (y kg per square metre) over several seasons at a farm. Some of their data is shown in the table and scatterplot below.

Rainfall, x	22	84	97	48	14	37	97	50	61	75	36	6
Yield, y	0.36	0.72	0.66	0.21	0.11	0.54	0.66	0.56	0.42	0.61	0.31	0.09
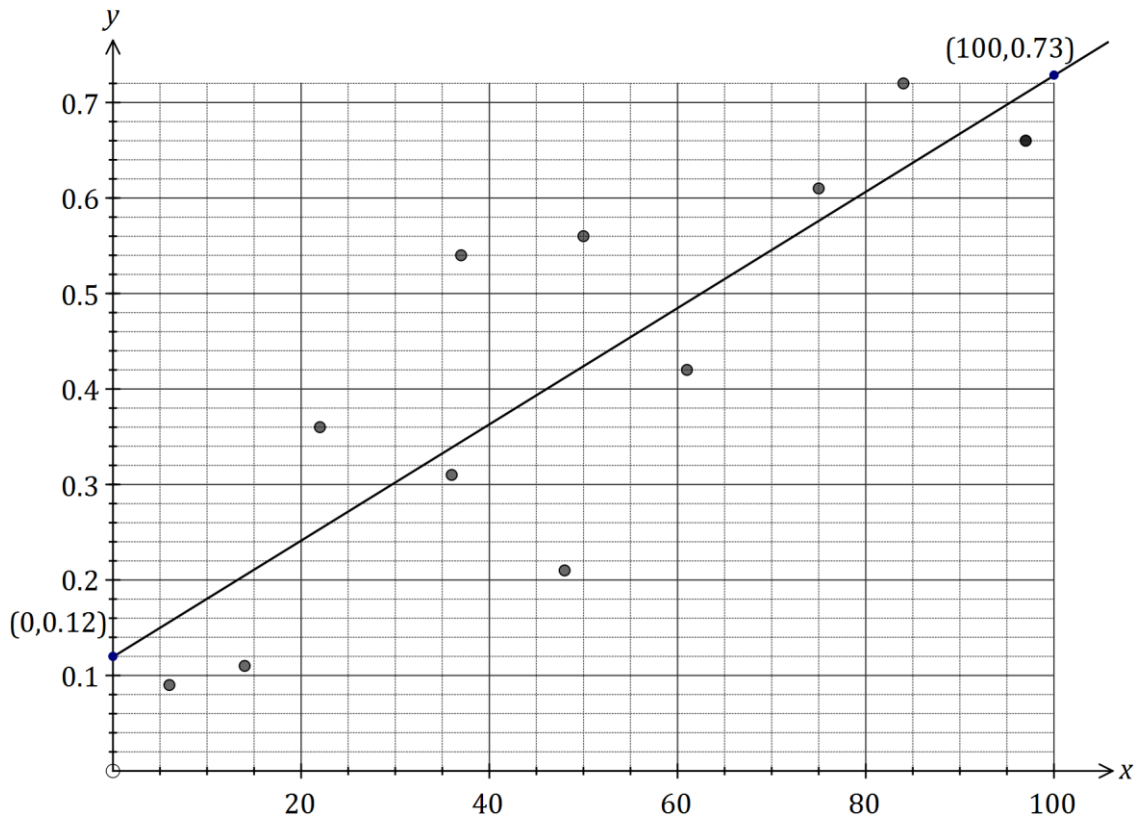
(a)   Calculate the correlation coefficient for the data, and comment on how its value is reflected in the scatterplot above.   (3 marks)

Solution
$r = 0.86$ Strong positive value is reflected in increasing trend shown by points.

Specific behaviours
✓ value that rounds to 0.86 ✓ indicates direction of association

(b)   What percentage of the variation in the yield can be explained by the variation in the rainfall?   (2 marks)

Solution
$r^2 = 0.735$ Approximately 74%.

Specific behaviours
✓ indicates use of coefficient of determination ✓ states correct percentage

See next page

(c) Determine the equation for the least-squares line that models the data. (2 marks)

Solution
$y = 0.00609x + 0.119$
Specific behaviours
✓ gradient
✓ y-intercept

(d) Draw the least-squares line on the scatterplot by first calculating two points that lie on the line. Clearly indicate these points. (3 marks)

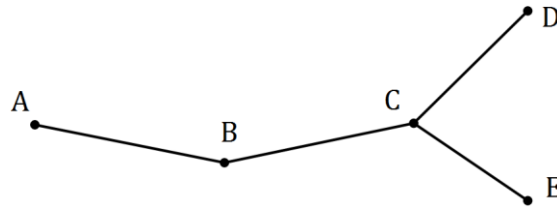| Solution |
|---|
| Examples: (0, 0.12) and (100, 0.73) |
| See graph for line. |
| **Specific behaviours** |
| ✓ clearly shows two points that are not close together |
| ✓ line through calculated points |
| ✓ correct line, very close to (0, 0.12) and (100, 0.73) |

(e) Estimate the cucumber yield in a season that has 64 mm of rainfall and comment on the reliability of this value. (3 marks)

Solution
$y = 0.00609(64) + 0.119 = 0.51 \text{ kg}$
Estimate is reliable as correlation is strong and involves interpolation.
Specific behaviours
✓ value that rounds to 0.51
✓ considers correlation
✓ considers interpolation

Question 10   **(8 marks)**

A simple connected graph is drawn below.

(a)   List, in order, the vertices of a closed walk on the graph of length 8 that visits all vertices
       and ends at A.

Solution
ABC DCE CBA or *ABC ECD CBA*

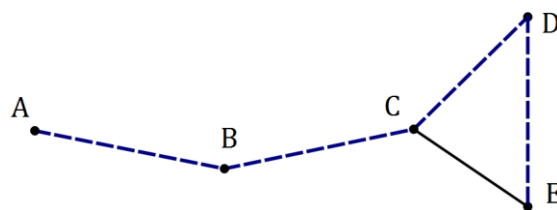| Specific behaviours |
| --- |
| ✓ starts and finishes at A |
| ✓ sequential walk of length 8 |

(2 marks)

(b)   Explain why the walk in (a) is not a Hamiltonian cycle.   (2 marks)

Solution
A Hamiltonian cycle is a closed path that visits every vertex other than start exactly once.
Walk in (a) visits several vertices more than once.

Specific behaviours
✓ indicates both properties of Hamiltonian cycle
✓ explains multiple vertices

(c)   Add one edge and one face to the copy of the graph below, so that the new graph
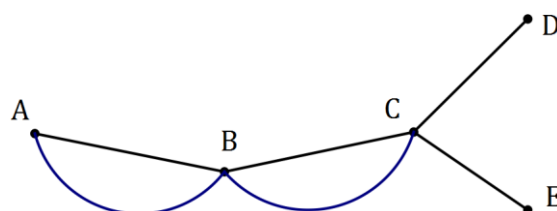   contains a Hamiltonian path and mark this path on the graph.   (2 marks)

Solution
See graph - many solns
Specific behaviours
✓ adds edge and face
✓ marks path

(d)   Add two edges and two faces to the copy of the graph below, so that the new graph does
       not contain a Hamiltonian path or cycle.   (2 marks)

Solution
See graph - many solns
Specific behaviours
✓ adds edges and faces
✓ still no HP or HC

Question 11   **(9 marks)**

The value of a machine used in a factory is recorded at the start of each year.

Year	2014	2015	2016
Value of machine ($)	6 875	5 500	4 400

(a)   Explain why the three values in the table form a geometric sequence.   (2 marks)

Solution
$\dfrac{5500}{6875} = 0.8, \dfrac{4400}{5500} = 0.8$ Hence geometric, as constant ratio of terms
Specific behaviours
✓ shows both pairs of terms have same ratio ✓ explains nature of geometric sequence

(b)   What is the annual percentage rate of depreciation of the machine?   (1 mark)

Solution
20%
Specific behaviours
✓ rate as percentage

Assume that the machine continues to depreciate at the same rate.

(c)   Determine a rule for V_n, the value of the machine n years after 2014.   (2 marks)

Solution
$V_n = 6875(0.8)^n$
Specific behaviours
✓ uses n^{th} term form ✓ uses correct coefficients

(d)   Determine the value of the machine at the start of the year 2020.   (2 marks)

Solution
$V_n = 6875(0.8)^6 = \$1\,802.24$
Specific behaviours
✓ substitutes correct value of n ✓ states value

(e)   The machine will be replaced when its value at the start of the year falls below $500. Determine which year this will be.   (2 marks)

Solution
$6875(0.8)^n = 500$ $n = 11.7 \approx 12$ Year 2026
Specific behaviours
✓ calculates n ✓ states year

Question 12   **(11 marks)**

In a recent study of artists who asked for a piece of their work to be included in an exhibition, each artist was classified by the variables (i) the state they worked in and (ii) whether their piece of work was accepted by the judges.

The table below shows the number of artists in each category.

	State	NSW	VIC	QLD	WA	Total
Work accepted?	Yes	8	27	21	8	64
	No	108	86	143	39	376
	Total	116	113	164	47	440

(a)   Complete the missing values and totals in the table above.   (4 marks)

Solution
See table
Specific behaviours
✓ QLD Yes, ✓ Total No, ✓ NSW No, ✓ Totals

(b)   To identify the presence of an association between these two variables, explain why the state the artist worked in should be used as the explanatory variable.   (2 marks)

Solution
It is possible for the 'state' to affect 'having work accepted', but not possible for 'having work accepted' to affect 'state artist works in'. Hence 'state' is explanatory variable and 'work accepted' is response variable.
Specific behaviours
✓ reasonable argument using just explanatory variable ✓ discussion using both explanatory and response variables

See next page

(c)   Rounding percentages to the nearest whole number, complete the percentaged two-way table below so that it may be used to identify the presence of an association between the categorical variables. (3 marks)

	State	NSW	VIC	QLD	WA
Work accepted?	Yes	7%	24%	13%	17%
	No	93%	76%	87%	83%

Solution
See table
Specific behaviours
✓ NSW Yes
✓ one column correct
✓ all columns correct

(d)   Comment on the presence of an association between the two variables. (2 marks)

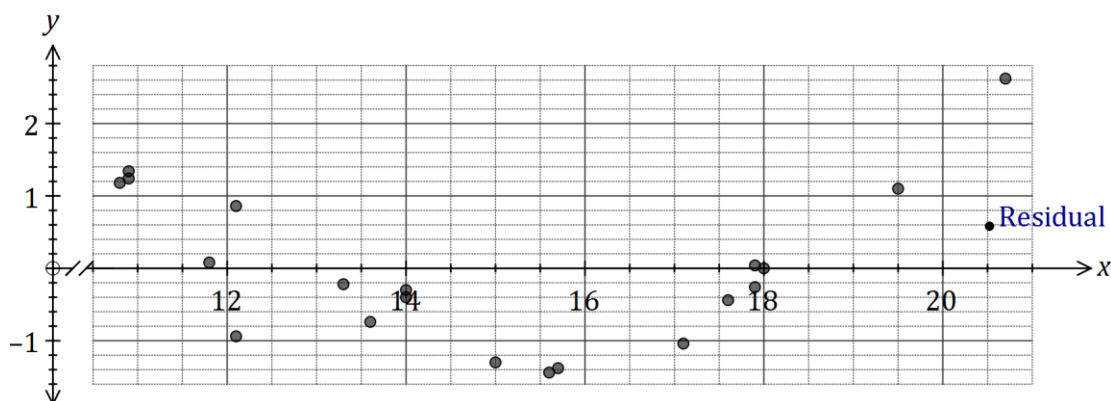| Solution |
|---|
| An association clearly exists, as artists from VIC (24%) are much more likely to have their work chosen than an artist from NSW (7%), QLD (13%) or WA (17%). |
| **Specific behaviours** |
| ✓ states association exists |
| ✓ explains reasoning |

Question 13   **(6 marks)**

A student was trying to decide whether fitting a linear model to their data was an appropriate choice. They calculated the least-squares line through the 30 points to be $\hat{y} = 2.4x - 12.5$ and the correlation coefficient $r = 0.98$.

(a)   Explain why constructing a residual plot would help the student decide.   (2 marks)

Solution
The residual plot will enable the student to see if any general pattern is evident in the residuals - if there is, the model will not be appropriate, but otherwise model is usually valid.
Specific behaviours
✓ indicates looking for pattern in residuals
✓ explains outcomes associated with pattern

The residual plot for the student's data is shown below.

(b)   One residual is missing from the plot, corresponding to the original data point $(20.5, 37.3)$.

        Calculate the residual for this point and add it to the residual plot.   (3 marks)

Solution
$\hat{y} = 2.4(20.5) - 12.5 = 36.7$
Residual $= 37.3 - 36.7 = 0.6$
Specific behaviours
✓ calculates $\hat{y}$
✓ calculates residual
✓ plots point

(c)   What conclusion should the student draw about the appropriateness of the linear model?

                                                       (1 mark)

Solution
Pattern evident so linear model not appropriate.
Specific behaviours
✓ states not appropriate

Question 14 **(10 marks)**

Following the analysis of data collected from a group of women aged between 22 and 38, a strong, linear relationship between their age (x, in years) and percentage chance of conception (y, percent) in any given month was observed. The coefficient of determination between the variables was 0.87 and the equation of the least-squares line was $\hat{y} = -0.88x + 45.8$.

(a) Determine the correlation coefficient between x and y. (2 marks)

Solution
$r^2 = 0.87 \Rightarrow r = \pm\sqrt{0.87} \approx \pm 0.93$
As slope negative, then $r = -0.93$
Specific behaviours
✓ determines value
✓ determines sign

(b) Estimate the monthly percentage chance of conception of a woman aged

(i) 18 years. (1 mark)

Solution
$\hat{y} = 30\%$
Specific behaviours
✓ states value

(ii) 35 years. (1 mark)

Solution
$\hat{y} = 15\%$
Specific behaviours
✓ states value

(c) Comment, with reasoning, on the reliability of each of your estimates in (b). (3 marks)

| Solution |
|---|
| (i) is unreliable, as involves extrapolation. |
| (ii) is reliable, as interpolation and correlation is strong. |
| **Specific behaviours** |
| ✓ indicates (i) unreliable (ii) reliable |
| ✓ reason for (i) |
| ✓ reason for (ii) |

(d) Describe the meaning of the slope of the least-squares lines in the context of this question. (2 marks)

Solution
For each year a woman in the group ages, so their percentage chance of conception decreases by 0.88%.
Specific behaviours
✓ indicates as age increases, % chance decreases
✓ quantifies change with units

Question 15   **(10 marks)**

The daily sales for a shoe store over three weeks are shown in the graph and tables below.
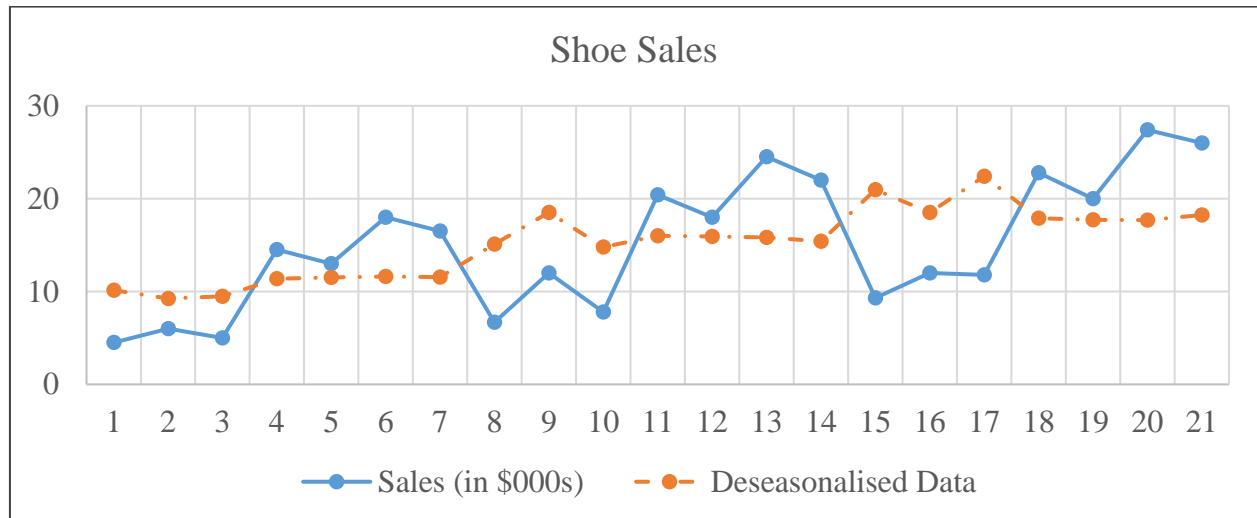
Week	Day	Time (t)	Sales (in $000s)	Cycle Mean	Seasonal Effect	Deseasonalised Data
1	Monday	1	4.5		40.65%	10.1
	Tuesday	2	6.0		54.19%	9.3
	Wednesday	3	5.0		45.16%	9.5
	Thursday	4	14.5	11.07	**A**	**B**
	Friday	5	13.0		117.42%	11.5
	Saturday	6	18.0		162.58%	11.6
	Sunday	7	16.5		149.03%	11.6
2	Monday	8	6.7		42.10%	15.1
	Tuesday	9	12.0		75.40%	18.5
	Wednesday	10	7.8		49.01%	14.8
	Thursday	11	20.4	15.91	128.19%	16.0
	Friday	12	18.0		113.11%	15.9
	Saturday	13	24.5		153.95%	15.8
	Sunday	14	22.0		138.24%	15.4
3	Monday	15	9.3		50.35%	21.0
	Tuesday	16	12.0		64.97%	18.5
	Wednesday	17	11.8		63.88%	22.4
	Thursday	18	22.8	18.47	123.43%	17.9
	Friday	19	20.0		108.28%	17.7
	Saturday	20	27.4		148.34%	17.7
	Sunday	21	26.0		140.76%	18.2

Seasonal Indices

Monday	Tuesday	Wednesday	Thursday	Saturday	Sunday
0.4436	0.6485	0.5269	1.2753	1.5496	**C**

(a) Describe any unseasonal fluctuations. (1 mark)

Solution
Tuesday week 2 (*t(9))* is unusually high

Specific behaviours
✓ indicates correct time period

(b) Calculate the missing values A, B and C in the tables on page 12.
(3 marks)

Solution
$A = 700\% - (40.65 + 54.19 + 45.16 + 117.42 + 162.58 + 149.03)$ $\quad = 130.97\%$ $B = 14.5 \div 1.2753 \qquad\qquad C = \dfrac{149.03 + 138.24 + 140.76}{3}$ $\quad = 11.369 \qquad\qquad\qquad\quad = 142.67\dot{6}\%$ $\quad \approx 11.4 \qquad\qquad\qquad\qquad\quad \approx 1.4268$

Specific behaviours
✓ correctly calculates A
✓ correctly calculates B
✓ correctly calculates C

The least squares regression line, using the deseasonalised data is $\hat{y} = 0.5221t + 9.4890$

(c) Using the line of regression, predict the sales for Wednesday of week 4 and comment on your prediction. (3 marks)

Solution
$t = 24$ $\hat{y} = 0.5221(21) + 9.4890$ $\quad = 22.0194$ $\hat{y} \times 0.5269 = 11.602$ $\quad\quad\quad \approx \$11\ 600$ Prediction is within 1 cycle of actual data, therefore reliable.

Specific behaviours
✓ indicates correct *t* value
✓ applies seasonal index
✓ states predication is reliable with justification

(d) The store will receive a bonus if during week 4 they can exceed $37 000 in sales on any day. If sales continue in the current trend, is the store likely to receive this bonus? Justify. (3 marks)

Solution
Saturday has the highest sales so use, *t* = 27. $t(27) = (0.5221(27) + 9.4890) \times 1.5496$ $\quad\quad = \$36\ 548$ They are not likely to reach the required, so will not receive bonus.

Specific behaviours
✓ predicts *t* = 27
✓ seasonalises prediction and coverts into thousands
✓ makes a comment based on result of prediction

Question 16   **(12 marks)**

As part of a trial to reintroduce woylies (an endangered species of mammals) to a wildlife reserve, researchers modelled the expected size of a woylie population, P_n, using the rule below where n is the number of months since the trial began.

$$P_{n+1} = 0.85P_n + 30, \qquad P_0 = 20.$$

(a)   State

       (i)   the size of the woylie population at the start of the trial.   (1 mark)

Solution
20 woylies
Specific behaviours
✓ states value

       (ii)   the number of woylies added to the reserve each month.   (1 mark)

Solution
30 woylies
Specific behaviours
✓ states value

       (iii)   the percentage loss of existing woylies in the reserve each month.   (1 mark)

Solution
15% loss
Specific behaviours
✓ states percentage

(b)   Complete the missing values in the table below to show the expected number of woylies over the first six months.   (2 marks)
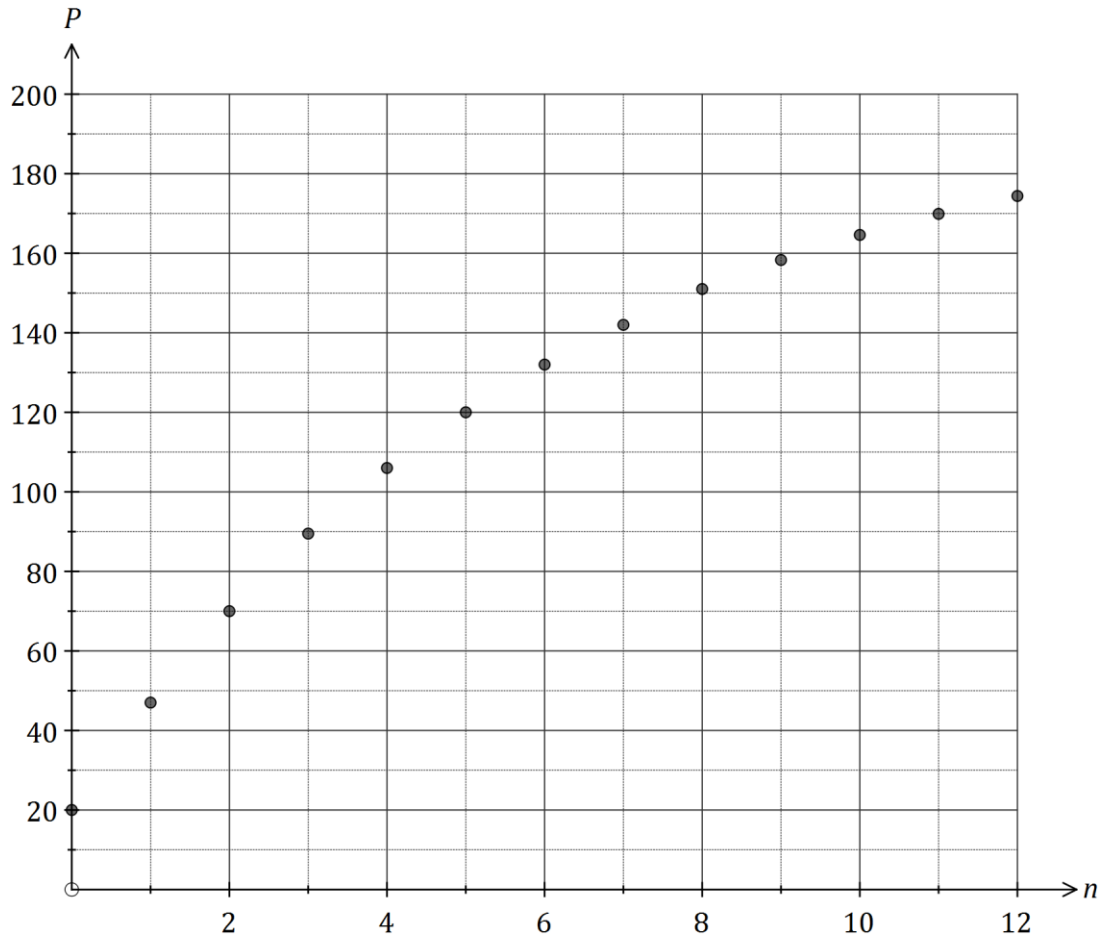
n	0	1	2	3	4	5	6
P_n	**20**	**47**	70	89	106	**120**	**132**

Solution
See table
Specific behaviours
✓ values for $n = 0, 1$
✓ values for $n = 5, 6$ rounded to whole number

(c)   Determine the expected size of the woylie population after three years.   (2 marks)

Solution
$P_{36} = 199$ woylies
Specific behaviours
✓ indicates use of $n = 36$
✓ states rounded value

(d)   Graph the population of woylies on the axes below for $0 \leq n \leq 12$.   (3 marks)

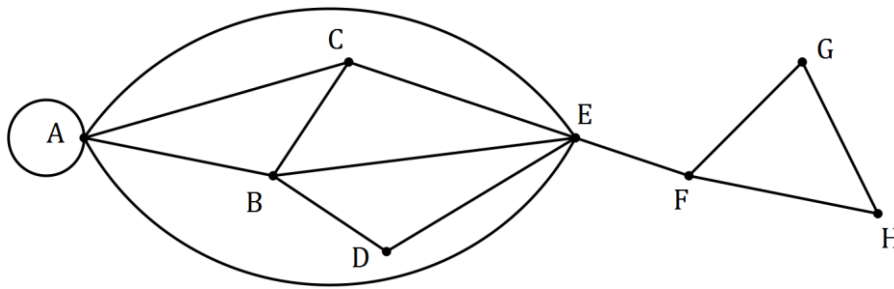| Solution |
|---|
| See graph |
| **Specific behaviours** |
| ✓ at least half of points accurately plotted |
| ✓ all points accurately plotted |
| ✓ just points, as not steady, continuous growth |

(e)   Use the model to describe how the size of the woylie population in the reserve will change over the first three years.   (2 marks)

Solution
Population will increase rapidly at the start, but then level out at close to 200 woylies after two years.
Specific behaviours
✓ indicates rapid increase at start
✓ indicates steady state after two years

Question 17   **(9 marks)**

The graph below represents 14 canals that meet at locations A to H.

(a)   A canal enthusiast noticed that the graph contained a semi-Eulerian trail. State the two
       properties of a walk that make a semi-Eulerian trail.   (2 marks)

Solution
Trail is open (start and end at different vertices) and includes every edge once.

Specific behaviours
✓ indicates open
✓ indicates includes all edges

(b)   What property of a connected graph indicates the existence of a Eulerian trail, a semi-
       Eulerian trail or neither?   (2 marks)

Solution
The number of odd vertices.
Eulerian has none, semi-Eulerian has 2, and neither has 4 or more.

Specific behaviours
✓ indicates number of odd vertices
✓ explains all three cases

(c)   Suggest a suitable starting point for the canal enthusiast to begin a semi-Eulerian trail and
       indicate where they will finish the trail.   (2 marks)

Solution
Start C, finish F or other way around.

Specific behaviours
✓ starting point
✓ finish point

(d)   Draw a subgraph of the above graph that is simple, connected, has no bridges and has 8
       edges.   (3 marks)

Solution

Specific behaviours
✓ removes RHS including bridge
✓ removes loop and one of edges AE
✓ subgraph with 8 edges

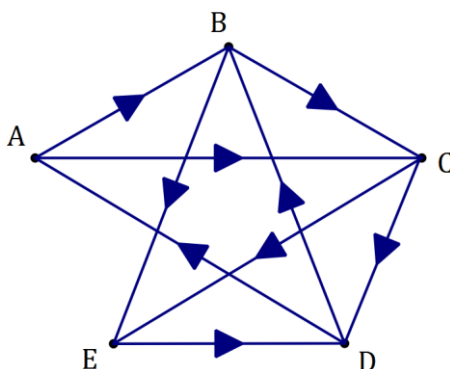**Question 18**   **(7 marks)**

Five children (A, B, C, D, E) are playing a game of hide and seek. The directed edges, shown in the adjacency matrix M below, represent whether the child in a row knows the location of the child in a column.

M	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	0	1
C	0	0	0	1	1
D	1	1	0	0	0
E	0	0	0	1	0

(a)   Construct a digraph to show the above information.   (3 marks)

Solution
See graph
Specific behaviours
✓ all edges
✓ at least 7 directions correct
✓ all directions correct

(b)   Given the two matrices:

$$M^2 = \begin{bmatrix} 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad M^2 + M = \begin{bmatrix} 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 2 & 2 \\ 1 & 1 & 0 & 2 & 1 \\ 1 & 2 & 2 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

(i)   If child D wanted to locate child E with the help of one other child, could they? Explain how an element of M^2 can help justify your answer.   (2 marks)

Solution
Yes.
$m_{45} > 0$ means they can find out through another child (B).
Specific behaviours
✓ states yes
✓ explanation involving m_{45}

(ii)   Two children are unable to locate all the other children, even with the help of one other child. Who are these two, and who can't they locate?   (2 marks)

Solution
B can't locate A
E can't locate C
(0 elements except diagonal in $M^2 + M$)
Specific behaviours
✓ identifies first pair
✓ identifies second pair